This contradicts the known results of Fermat, and our proof is complete.

It is well known that the equation $X^4 + Y^4 = Z^2$ *has no nonzero integer solutions. The proof of this result by Fermat is based on his method of infinite descent. Using exactly the same argument, it can be shown that the equation* $X^4 - Y^4 = Z^2$ *has no nonzero integer solutions. For example, see Theorem 13.3 on pages 520–522 and Exercise No. 4 on p. 525 of the book* Elementary Number Theory and its Applications, *5th edition, by Kenneth Rosen.*

3450. [2009 : 235, 237] *Proposed by Dragoljub Milošević, Gornji Milanovac, Serbia.*

Let $\triangle ABC$ have inradius r, exradii r_a, r_b, r_c, and altitudes h_a, h_b, h_c. Prove that

$$\frac{h_a + 2r_a}{r + r_a} + \frac{h_b + 2r_b}{r + r_b} + \frac{h_c + 2r_c}{r + r_c} \geq \frac{27}{4}.$$

Solution by Arkady Alt, San Jose, CA, USA; Dung Nguyen Manh, Student, Hanoi University of Technology, Hanoi, Vietnam; Thanos Magkos, 3rd High School of Kozani, Kozani, Greece; and Panos E. Tsaoussoglou, Athens, Greece, independently.

Let a, b, c be the sides, A the area, and s the semiperimeter of the triangle ABC. We have

$$\begin{aligned} \sum_{\text{cyclic}} \frac{h_a + 2r_a}{r + r_a} &= \sum_{\text{cyclic}} \frac{\left(\dfrac{2A}{a} + \dfrac{2A}{s-a}\right)}{\left(\dfrac{A}{s} + \dfrac{A}{s-a}\right)} \\ &= \sum_{\text{cyclic}} \frac{2s^2}{a(2s-a)} = \sum_{\text{cyclic}} \frac{(a+b+c)^2}{2a(b+c)}. \end{aligned}$$

Using the well-known and easy to prove inequality

$$(a+b+c)^2 \geq 3(ab+bc+ca)$$

and the Cauchy–Schwarz inequality, we obtain

$$\begin{aligned} \sum_{\text{cyclic}} \frac{(a+b+c)^2}{2a(b+c)} &\geq \sum_{\text{cyclic}} \frac{3(ab+bc+ca)}{2a(b+c)} \\ &= \frac{3}{2}(ab+bc+ca) \sum_{\text{cyclic}} \frac{1}{a(b+c)} \\ &= \frac{3}{4}\left(\sum_{\text{cyclic}} a(b+c)\right)\left(\sum_{\text{cyclic}} \frac{1}{a(b+c)}\right) \\ &\geq \frac{3}{4}(1+1+1)^2 = \frac{27}{4}, \end{aligned}$$

as claimed.

Also solved by GEORGE APOSTOLOPOULOS, Messolonghi, Greece; ŠEFKET ARSLANAGIĆ, University of Sarajevo, Sarajevo, Bosnia and Herzegovina; MICHEL BATAILLE, Rouen, France; CHIP CURTIS, Missouri Southern State University, Joplin, MO, USA; OLIVER GEUPEL, Brühl, NRW, Germany; JOE HOWARD, Portales, NM, USA; HUNEDOARA PROBLEM SOLVING GROUP, Hunedoara, Romania; WALTHER JANOUS, Ursulinengymnasium, Innsbruck, Austria; WEI-DONG, Weihai Vocational College, Weihai, Shandong Province, China; KEE-WAI LAU, Hong Kong, China; SALEM MALIKIĆ, student, Sarajevo College, Sarajevo, Bosnia and Herzegovina; CRISTINEL MORTICI, Valahia University of Târgoviște, Romania; PETER Y. WOO, Biola University, La Mirada, CA, USA; TITU ZVONARU, Comănești, Romania; and the proposer.